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Math(s) = Maths!

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<http://www.seab.gov.sg/oLevel/oLevel.html>

Editor: Chad Evans, www.knowneedlah.com , www.facebook.com/KnowNeedLah

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Numbers

Types of numbers

Integers: are whole numbers (both positive and negative). Zero is usually classed as an integer. Natural numbers are positive integers.

A rational : number is a number which can be written as a fraction where numerator and denominator are integers. For example $\frac{1}{2}$, 4, $1.75 (= \frac{7}{4})$. A rational number can be written as an exact or recurring decimal. For example 0.175 is rational since it is an exact decimal. $0.345345345\dots$ is rational since it is a recurring decimal.

Irrational : numbers are numbers which cannot be written as fractions, eg π , $\sqrt{2}$.

Square numbers : are numbers which can be obtained by multiplying another number by itself. E.g. 36 is a square number because it is 6×6 .

Surds: are numbers left written as \sqrt{n} , where n is positive but not a square number. E.g. $\sqrt{2}$.

Prime: numbers are numbers above 1 which cannot be divided by anything, other than 1 and itself, to give an integer. Examples: 2, 3, 5, 7, 11, 13, 17, 19.

Real : numbers are all the numbers which you will have come across (i.e. all the rational and irrational numbers).

LCM and HCF

The lowest common multiple (LCM) of two or more numbers is the smallest number into which they both evenly divide. For example, the LCM of 2, 3, 4, 6 and 9 is 36.

The highest common factor (HCF) of two or more numbers is the highest number which will divide into them both. Therefore the HCF of 6 and 9 is 3.

Rounding Numbers

If the answer to a question was 0.00256023164, you would not write this down. Instead, you would 'round off' the answer. There are two ways to do this, you can round off to a certain number of decimal places or a certain number of significant figures.

The above number, rounded off to 5 decimal places (d.p.) is 0.00256. You write down the 5 numbers after the decimal point. To round the number to 5 significant figures, you write down 5 numbers.

However, you do not count any zeros at the beginning. So to 5 s.f. (significant figures), the number is 0.0025602 (5 numbers after the first non-zero number appears).

From what we know so far, if you rounded 4.909 to 2 decimal places, the answer would be 4.90. However, the number is closer to 4.91 than 4.90, because the next number is a 9. Therefore, the rule is: if you are rounding a number, if the number after the place you stop is 5 or above, you add one to the last number you write.

So 3.486 to 3s.f. is 3.49

0.0096 to 3d.p. is 0.010 (This is because you add 1 to the 9, making it 10.

When rounding to a number of decimal places, always write any zeros at the end of the number. If you say 3d.p., write 3 decimal places, even if the last digit is a zero).

Approximations

If the side of a square field is given as 90m, correct to the nearest 10m:

The smallest value the actual length could be is 85m (since this is the lowest value which, to the nearest 10m, would be rounded up to 90m). The largest value is 95m.

Using inequalities, $85 > \text{length} < 95$.

Sometimes you will be asked the upper and lower bounds of the area. The area will be smallest when the side of the square is 85m. In this case, the area will be 7725m^2 . The largest possible area is 9025m^2 (when the length of the sides are 95m).

BODMAS

When simplifying an expression such as $3 + 4 \times 5 - 4(3 + 2)$, remember to work it out in the following order: brackets, of, division, multiplication, addition, subtraction.

So do the thing in the brackets first, then any division, followed by multiplication and so on. The above is: $3 + 20 - 4 \times 5 = 3 + 20 - 20 = 3$

Directed Numbers

Numbers can either be positive and negative. Often brackets are put around negative numbers to make them easier to read, e.g. (-2). If a number is positive, the + is usually missed out before the number. So 3 is really (+3). Adding and multiplying combinations of positive and negative numbers can cause confusion and so care must be taken.

Addition and Subtraction

Two 'pluses' make a plus, two 'minuses' make a plus. A plus and a minus make a minus.

Example:

$$3 + (-2)$$

A plus and a minus make a minus, so this is the same as $3 - 2 = 1$

Example:

$$(-2) + (-5) = -7$$

This is the same as $(-2) - 5 = -7$

Multiplication and Division

If two positive numbers are multiplied together or divided, the answer is positive.

If two negative numbers are multiplied together or divided, the answer is positive.

If a positive and a negative number are multiplied or divided, the answer is negative.

Examples:

$$(-2) \div (-4) = \frac{1}{2}$$

$$(8) \div (-2) = (-4)$$

$$2 \times (-3) = (-6)$$

$$(-2) \times (-2) = 4$$

Fractions

Introduction

$1/2$ means 1 divided by 2. If you try this on a calculator, you will get an answer of 0.5. $3/6$ means 3 divided by 6. Using a calculator, you will find that this too gives an answer of 0.5. That is because $1/2 = 3/6 = 0.5$. Fractions such as $3/6$ can be cancelled. You can divide the top and bottom of the fraction by 3 to get $1/2$.

With fractions, you are allowed to multiply or divide the top and bottom of the fraction by some number, as long as you multiply (/divide) everything on the top and everything on the bottom by that number.

So $5/12 = 10/24$ (multiplying top and bottom by 2).

Adding and subtracting fractions

To add two fractions, the bottom (denominator) of the two fractions must be the same. $1/2 + 3/2 = 4/2$; $1/10 + 3/10 + 5/10 = 9/10$. If they are not the same, multiply or divide the top and bottom of one of the fractions by a number to make the denominator the same as the other.

Example:

$$\frac{5}{6} + \frac{2}{3} = \frac{5}{6} + \frac{4}{6} = \frac{9}{6} = \frac{3}{2}$$

The same is true when subtracting fractions.

Multiplying fractions

This is simple, just multiply the two numerators (top bits) together, and the two denominators together:

$$\frac{2}{3} \times \frac{5}{8} = \frac{10}{24} = \frac{5}{12}$$

Dividing Fractions

If A, B, C and D are any numbers,

$$\frac{A}{B} \text{ divided by } \frac{C}{D} = \frac{A}{B} \text{ multiplied by } \frac{D}{C}$$

So:

$$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

Harder examples

These rules work even when the fractions involve algebra.

$$\frac{2x}{5} \div \frac{3}{x} = \frac{6x}{5x} = \frac{6}{5} \quad (\text{the } x\text{'s cancel})$$

(See algebraic fractions section for harder examples)

A note on canceling

Fractions, of course, can often be 'cancelled down' to make them simpler. For example, $4/6 = 2/3$. You can divide or multiply the top and bottom of any fraction by any number, as long as you do it to both the top and bottom. However, when there is more than one term on the top and/or bottom, to cancel you must divide every term in the top and bottom by that number.

Examples:

$$\frac{2 + x}{2}$$

In this example, some people might try to cancel the 2s, but you cannot do this. You would have to divide the x by 2 also, to get $1 + \frac{1}{2}x$.

$$\frac{2(x + 4)}{4} = \frac{(x + 4)}{2}$$

Here there is only one term in the numerator (top) and denominator (bottom) of the fraction, so you can divide top and bottom by 2.

Example:

Simplify the expression $\frac{2x^2 - 5x + 2}{x^2 - 4}$

In questions such as this, it is often useful to factorise.

$$\frac{(2x - 1)(x - 2)}{(x + 2)(x - 2)}$$

Factorising means that there is now only one term in the numerator and denominator, whereas before there were two. We can now divide top and bottom by $(x - 2)$:

$$\frac{(2x - 1)}{(x + 2)}$$

Surds

Basic surd manipulation

Surds are numbers left in 'square root form' (or 'cube root form' etc).

Addition and subtraction of surds:

$$4\sqrt{7} - 2\sqrt{7} = 2\sqrt{7}.$$

$$5\sqrt{2} + 8\sqrt{2} = 13\sqrt{2}$$

Note: $5\sqrt{2} + 3\sqrt{3}$ cannot be manipulated because the surds are different (one is $\sqrt{2}$ and one is $\sqrt{3}$).

Multiplication:

$$\sqrt{5} \times \sqrt{15} = \sqrt{75} (= 15 \times 5)$$

$$= \sqrt{25} \times \sqrt{3}$$

$$= 5\sqrt{3}.$$

$$(1 + \sqrt{3}) \times (2 - \sqrt{8}) \quad [\text{The brackets are expanded as usual}]$$

$$= 2 - \sqrt{8} + 2\sqrt{3} - \sqrt{24}$$

$$= 2 - 2\sqrt{2} + 2\sqrt{3} - 2\sqrt{6}$$

Rationalising the denominator:

It is untidy to have a fraction which has a surd denominator. This can be 'tidied up' by multiplying the top and bottom of the fraction by a surd. This is known as rationalising the denominator, since surds are irrational numbers and so you are changing the denominator from an irrational to a rational number.

Example:

Rationalise the denominator of:

a) $\frac{1}{\sqrt{2}}$.

b) $\frac{1+2}{1-\sqrt{2}}$

a) Multiply the top and bottom of the fraction by $\sqrt{2}$. The top will become $\sqrt{2}$ and the bottom will become 2 ($\sqrt{2}$ times $\sqrt{2} = 2$).

b) In situations like this, look at the bottom of the fraction (the denominator) and change the sign (in this case change the plus into minus). Now multiply

the top and bottom of the fraction by this.

Therefore:

$$\frac{1+2}{1-\sqrt{2}} = \frac{(1+2)(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} = \frac{1+\sqrt{2}+2+2\sqrt{2}}{1+\sqrt{2}-\sqrt{2}-2} = \frac{3+3\sqrt{2}}{-1}$$

$$= -3(1+\sqrt{2})$$

Percentages

Percentages

A percentage is a fraction whose denominator is 100 (the numerator of a fraction is the top term, the denominator is the bottom term).

So $30\% = 30/100 = 3/10 = 0.3$

To change a decimal into a percentage, multiply by 100. So $0.3 = 0.3 \times 100 = 30\%$.

Example:

Find 25% of 10 (remember 'of' means 'times').

$\frac{25}{100} \times 10$ (divide by 100 to convert the percentage to a decimal)

$$= 2.5$$

Percentage Change

$$\% \text{ change} = \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100$$

$$\% \text{ error} = \frac{\text{error}}{\text{real value}} \times 100$$

Original value

$$\text{Original cost} = \frac{\text{New cost}}{100 + \% \text{ profit}} \times 100$$

Example:

A dealer buys a stamp collection and sells it for S\$2700, making a 35% profit. Find the cost of the collection.

It is the original cost/ value we wish to find, so the last formula is used.

$$\frac{2700}{100 + 35} \times 100 = \text{S\$}2000$$

Percentage Increases and Interest

$$\text{New value} = \frac{100 + \text{percentage increase}}{100} \times \text{original value}$$

Example:

S\$500 is put in a bank where there is 6% per annum interest. Work out the amount in the bank after 1 year.

In other words, the old value is S\$500 and it has been increased by 6%.

Therefore, new value = $106/100 \times 500 = S\$530$.

If in this example, the money was left in the bank for another year, the £530 would increase by 6%. The interest, therefore, will be higher than the previous year. This phenomenon is known as compound interest.

The simple way to work out compound interest is to multiply the money that was put in the bank by n to the power m , where n is $(100 + \text{percentage increase})/100$ and m is the number of years the money is in the bank for.

So if the S\$500 had been left in the bank for 9 years, the amount would have increased to:

$$500 \times (1.06)^9 = S\$845$$

Percentage decreases:

New value = $\frac{100 - \text{percentage decrease}}{100} \times \text{original value}$

Example:

At the end of 1993 there were 5000 members of a certain rare breed of animal remaining in the world. It is predicted that their number will decrease by 12% each year. How many will be left at the end of 1995?

At the end of 1994, there will be $(100 - 12)/100 \times 5000 = 4400$

At the end of 1995, there will be $88/100 \times 4400 = 3872$

Number Sequences

In the sequence 2, 4, 6, 8, 10... there is an obvious pattern. Such sequences can be expressed in terms of the n th term of the sequence. In this case, the n th term = $2n$. To find the 1st term, put $n = 1$ into the formula, to find the 4th term, replace the n 's by 4's: 4th term = $2 \times 4 = 8$.

Example:

What is the n th term of the sequence 2, 5, 10, 17, 26... ?

n	=	1	2	3	4	5
n^2	=	1	4	9	16	25
$n^2 + 1$	=	2	5	10	17	26

This is the required sequence, so the n th term is $n^2 + 1$. There is no easy way of working out the n th term of a sequence, other than to try different possibilities.

Tips: if the sequence is going up in threes (e.g. 3, 6, 9, 12...), there will probably be a three in the formula, etc.

In many cases, square numbers will come up, so try squaring n , as above.

Also, the triangular numbers formula often comes up. This is $n(n + 1)/2$.

Example:

Find the n th term of the sequence: 2, 6, 12, 20, 30...

n	=	1	2	3	4	5
$n(n + 1)/2$	=	1	3	6	10	15

Clearly the required sequence is double the one we have found the n th term for, therefore the n th term of the required sequence is $2n(n+1)/2 = n(n + 1)$.

The Fibonacci sequence is an important sequence which is as follows: 1, 1, 2, 3, 5, 8, 13, 21, The next term of this well-known sequence is found by adding together the two previous terms.

Standard Form

Standard Form

Standard form is a way of writing down very large or very small numbers easily. $10^3 = 1000$, so $4 \times 10^3 = 4000$. So 4000 can be written as 4×10^3 . This idea can be used to write even larger numbers down easily in standard form.

Small numbers can also be written in standard form. However, instead of the index being positive (in the above example, the index was 3), it will be negative.

The rules when writing a number in standard form is that first you write down a number between 1 and 10, then you write $\times 10$ (to the power of a number).

Example: Write 81 900 000 000 000 in standard form:

$$81\,900\,000\,000\,000$$

$$= 8.19 \times 10^{13}$$

It's 10^{13} because the decimal point has been moved 13 places to the left to get the number to be 8.19.

Example: Write 0.000 001 2 in standard form:

$$0.000\,001\,2 = 1.2 \times 10^{-6}$$

It's 10^{-6} because the decimal point has been moved 6 places to the right to get the number to be 1.2.

On a calculator, you usually enter a number in standard form as follows:
Type in the first number (the one between 1 and 10). Press EXP . Type in the power to which the 10 is risen.

Manipulation in Standard Form

This is best explained with an example:

The number p written in standard form is 8×10^5 .
The number q written in standard form is 5×10^{-2} .

Calculate $p \times q$. Give your answer in standard form.

Multiply the two first bits of the numbers together and the two second bits together:

$$\begin{aligned} & 8 \times 5 \times 10^5 \times 10^{-2} \\ & = 40 \times 10^3. \text{ Remember } 10^5 \times 10^{-2} = 10^3 \text{ (add the powers when multiplying)} \end{aligned}$$

The question asks for the answer in standard form, but this is not in standard form because the first part (the 40) should be a number between 1 and 10.

$$= \underline{\underline{4 \times 10^4}}$$

Calculate $p \div q$. Give your answer in standard form.

This time, divide the two first bits of the standard forms. Divide the two second bits. Multiply these two together.

$$\begin{aligned} & (8 \div 5) \times (10^5 \div 10^{-2}) \\ & = \underline{\underline{1.6 \times 10^7}} \end{aligned}$$

Ratios

Ratios

If the ratio of one length to another is 1 : 2, this means that the second length is twice as large as the first. If a boy has 5 sweets and a girl has 3, the ratio of the boy's sweets to the girl's sweets is 5 : 3. The boy has $\frac{5}{3}$ times more sweets as the girl, and the girl has $\frac{3}{5}$ as many sweets as the boy. Ratios behave much like fractions and can be simplified.

Example:

Simone made a scale model of a 'hot rod' car on a scale of 1 to 12.5. The height of the model car is 10cm.

(a) Work out the height of the real car.

The ratio of the lengths is 1 : 12.5. So for every 1 unit of length the small car is, the real car is 12.5 units. So if the small car is 10 units long, the real car is 125 units long. If the small car is 10cm high, the real car is 125cm high.

(b) The length of the real car is 500cm. Work out the length of the model car.

We know that model : real = 1 : 12.5. However, the real car is 500cm, so 1 : 12.5 = x : 500 (the ratios have to remain the same). x is the length of the model car. To work out the answer, we convert the ratios into fractions:

$$\frac{1}{12.5} = \frac{x}{500}$$

multiply both sides by 500:

$$500/12.5 = x$$

$$\text{so } x = 40\text{cm}$$

Example:

Bob and Sarah divide S\$40 in the ratio 3 : 5. How much do they each get?

First, add up the two numbers in the ratio to get 8. Next divide the total amount by 8, i.e. divide S\$40 by 8 to get S\$5. S\$5 is the amount of each 'unit' in the ratio. To find out how much Bob gets, multiply S\$5 by 3 ('units') = S\$15. To find out how much Sarah gets, multiply S\$5 by 5 = S\$25.

Variation

Proportion

If a is proportional to b,

$$a \propto b$$

and $a = kb$, where k is a constant

The value of k will be the same for all values of a and b.

Example:

If $a \propto b$, and $b = 10$ when $a = 5$, find an equation connecting a and b.

$$a = kb \quad (1)$$

Substitute the values of 5 and 10 into the equation to find k:

$$5 = 10k$$

$$\text{so } k = 1/2$$

substitute this into (1)

$$a = \frac{1}{2}b$$

Similarly, if m is proportional to n^2 , $m = kn^2$

Inverse Proportion

If a and b are inversely proportional to one another,

$$a \propto 1/b$$

$$a = k/b$$

In these examples, k is known as the constant of variation.

Example:

If b is inversely proportional to the square of a, and when $a = 3$, $b = 1$, find the constant of variation.

$$b = k/a^2$$

$$\text{when } a = 3, b = 1$$

$$1 = k/3^2$$

$$k = 9$$