'O' Level Revision Notes - Know Need Lah!
Module 2: Shape and Space

Angles, Shapes, Areas and Volumes, Circle Theorems,
Loci, Vectors and Transformations.

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Contents and Instructions

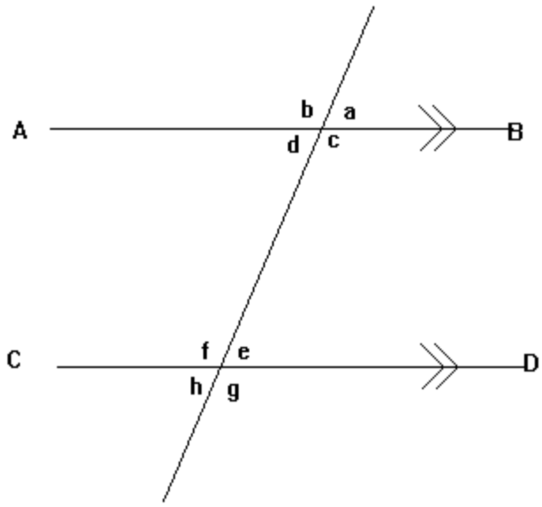
Math(s) = Maths!

If you are in Singapore use may want to use this links to view your syllabus.
http://www.seab.gov.sg/oLevel/oLevel.html

Editor: Chad Evans, www.knowneedlah.com , www.facebook.com/KnowNeedLah

Angles

Related Angles



Lines AB and CD are parallel to one another (hence the » on the lines).
a and d are vertically opposite angles. Vertically opposite angles are equal. (b and c, e and h, f and g are also vertically opposite).
g and c are corresponding angles. Corresponding angles are equal. (h and d, f and b, e and a are also corresponding).
d and e are alternate angles. Alternate angles are equal. (c and f are also alternate).
Alternate angles form a 'Z' shape and are sometimes called 'Z angles'.
a and b are adjacent angles. Adjacent angles add up to 180 degrees. (d and c, c and a, d and b, f and e, e and g, h and g, h and f are also adjacent).
d and f are interior angles. These add up to 180 degrees (e and c are also interior).

Any two angles that add up to 180 degrees are known as supplementary angles.

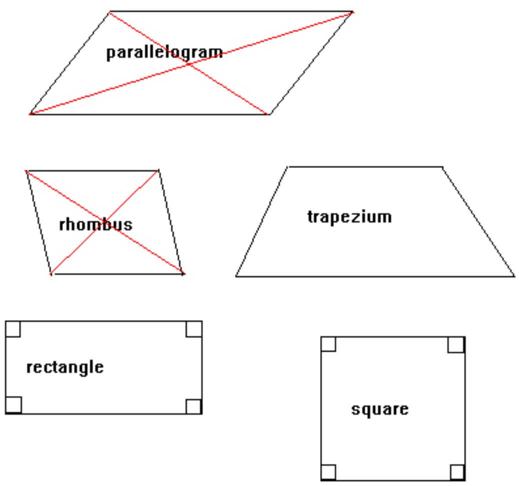
The angles around a point add up to 360 degrees.
The angles in a triangle add up to 180 degrees.
The angles in a quadrilateral add up to 360 degrees.
The angles in a polygon (a shape with n sides) add up to 180(n - 2) degrees.
The exterior angles of any polygon add up to 360 degrees.

Shapes

Symmetry
If a shape has a line of symmetry, the line of symmetry will divide the shape into two equal parts, one half of which can be folded along the line of symmetry to fit exactly onto the other. Note, a rectangle has two (not four) lines of symmetry and a circle has an infinite number.
If rotating a shape through a certain angle produces an identical shape, it has rotational symmetry. If the shape can be rotated 4 times before returning to its original shape (e.g. a square), it has rotational symmetry of order 4. An equilateral triangle has rotational symmetry of order 3 and a rectangle of order 2.

Triangles
Isosceles triangles have two equal angles. The sides of the triangle opposite the angles are equal to one another.
Equilateral triangles have all of their sides and angles equal. Since there are 180 degrees in a triangle and all the angles are equal, each angle must be 60 degrees.

Other Shapes



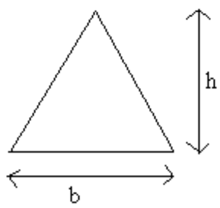
Parallelogram: opposite sides are parallel, opposite angles are equal, the diagonals bisect one another.
Rhombus: (a parallelogram with all four sides of equal length), diagonal bisect one another at right angles.
Trapezium: One pair of opposite sides are parallel.
Square: All sides are equal, all angles are 90 degrees, diagonals bisect one another at 90 degrees.
Rectangle: All angles are 90 degrees, diagonals bisect one another.

Areas and Volumes

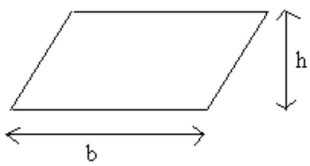
Areas and Volumes
*** Remember, with many exam boards, formulae will be given to you in the exam. However, you need to know how to apply the formulae and knowing them (especially the simpler ones) will help you in the exam. ***

A prism is a shape with a constant cross section (examples: cylinder, cuboid).
The volume of a prism = the area of the base × the length.

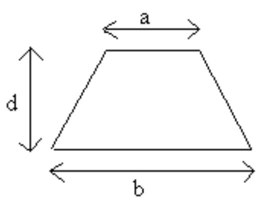
Areas (see shapes):
The area of a triangle = half × base × perpendicular height
The area of a circle = pr^2 (r is the radius of the circle)
The area of a parallelogram = base × perpendicular height
Area of a trapezium = half × (sum of the parallel sides) × the distance between them [1/2(a+b)d]



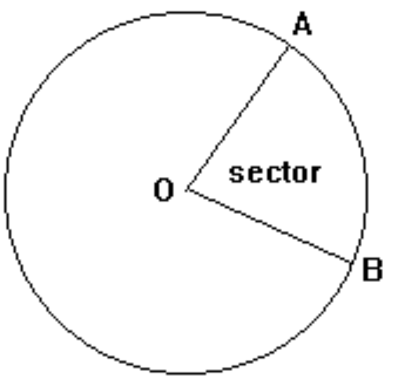
Area of triangle: ½bh



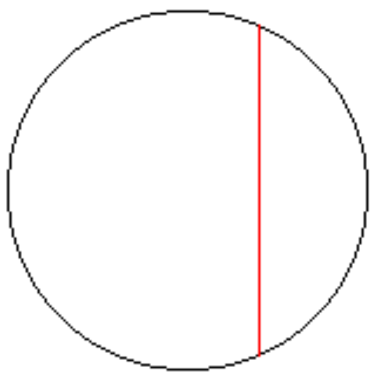
Area of parallelogram = bh



Area of trapezium = ½ (a + b)d



AB is part of the circumference and is known as an arc



Spheres:
Volume: $4/3pr^3$
Surface area: $4pr^2$

Cylinder:
Curved surface area: 2prh
Volume: pr^2h

Pyramid:
Volume = 1/3 × area of base × perpendicular height (=$1/3pr^2h$ for circular based pyramid).

Cone:
Curved surface area: prl (l is slant height)
Volume: $1/3pr^2h$ (h is perpendicular height)

WHEN USING FORMULAE FOR AREA AND VOLUME IT IS NECESSARY THAT ALL MEASUREMENTS ARE IN THE SAME UNITS.

Units
1 kilometre (km) = 1000 m
1 metre (m) = 100cm
1 centimetre (cm) = 10mm
1 litre = 1000 cm^3
1 hectare = 10 000 m^2
1 kilogram (kg) = 1000g (grams)

When working with lengths try to use metres if possible and when working with mass, use kilograms.

$1cm^2 = 100mm^2$ (10mm × 10mm)
$1cm^3 = 1000mm^3$ (10mm × 10mm × 10mm)

Ratios of lengths, areas and volumes
Imagine two squares, one with sides of length 3cm and one with sides of length 6cm. The ratio of these lengths is 3 : 6 (= 1 : 2). The area of the first is 9cm and the area of the second is 36cm. The ratio of these areas is 9 : 36 (= 1 : 4) .
In general, if the ratio of two lengths (of similar shapes) is a : b, the ratio of their areas is $a^2 : b^2$. The ratio of their volumes is $a^3 : b^3$.
This is why the ratio of the length of a mm to a cm is 1:10 (there are 10mm in a cm). The ratio of their areas (i.e. mm^2 to cm^2) is $1:10^2$ (there are 100mm^2 in a cm^2) and the ratio of their volumes (mm^3 to cm^3) is $1:10^3$ (there are 1000mm^2 in a cm^2).

Dimensions

Lines have one dimension, areas have two dimensions and volumes have three. Therefore if you are asked to choose a formula for the volume of an object from a list, you will know that it is the one with three dimensions.

Example:

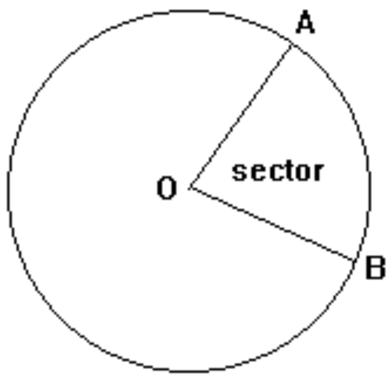
The letters r, l, a and b represent lengths. From the following, tick the three which represent volumes.

pr^2l
$2pr^2$
$4pr^3$
abrl
abl/r
$3(a^2 + b^2)r$
prl

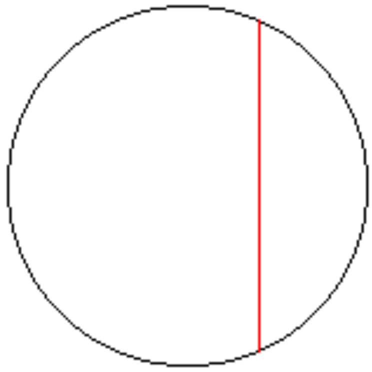
NB: Numbers are dimensionless so ignore p, 2, 4 and 3.
The first has three dimensions, since it is r × r × l.
The second has two dimensions (r × r).
The third has three dimensions (r × r × r).
etc.
$3(a^2 + b^2)r$ is the third formula with three dimensions. The expanded version of this formula is $3a^2r + 3b^2r$ and 3 dimensions + 3 dimensions = 3 dimensions (the dimension can only be increased or reduced by multiplication or division).

Circle Theorems

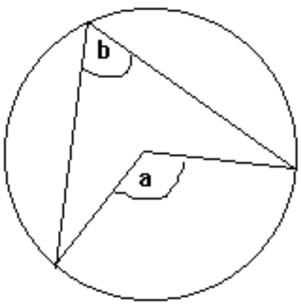
Circles

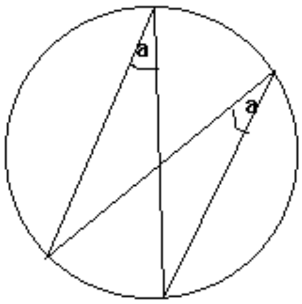


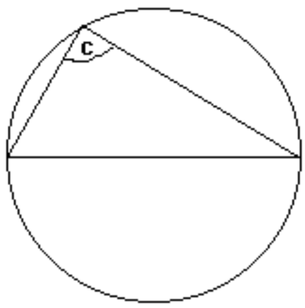
AB is part of the circumference
and is known as an arc

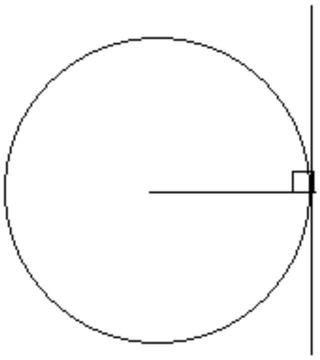


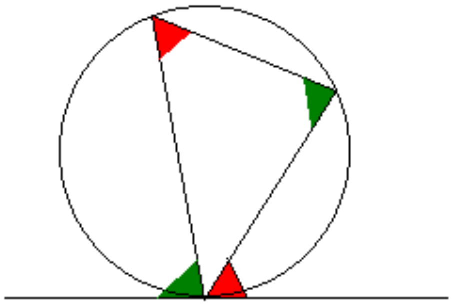
The red line in the second diagram is called a chord. It divides the circle into a major segment and a minor segment.







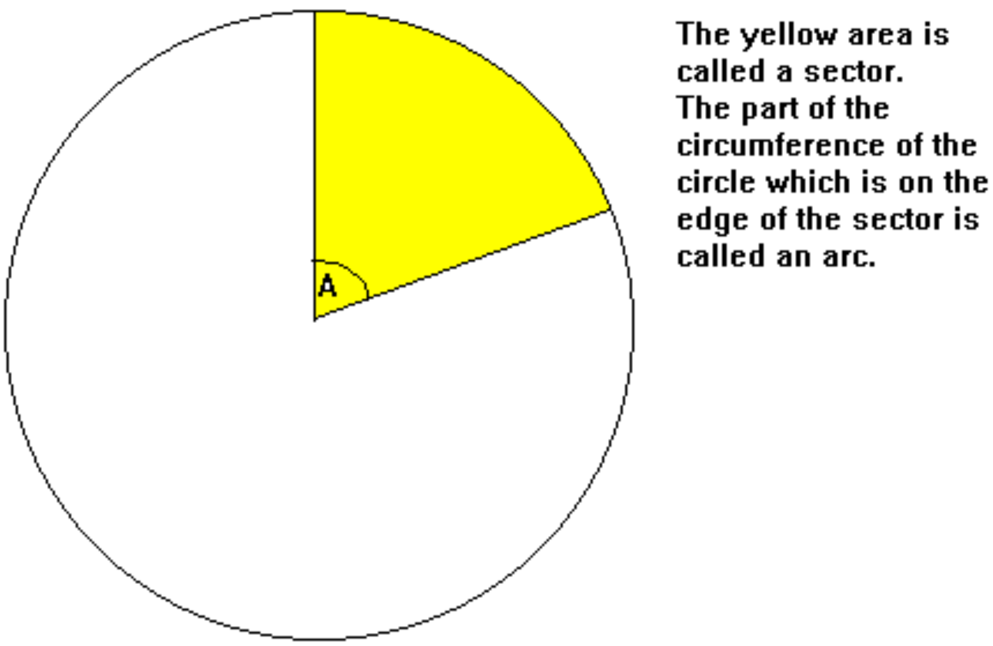




The diagrams show that:
The angle formed at the centre of the circle by lines originating from two points on the circle's circumference is double the angle formed on the circumference of the circle by lines originating from the same points. i.e. in the first diagram, $a = 2b$.
Angles formed from two points on the circumference are equal to other angles, in the same arc, formed from those two points.
Angles formed by drawing lines from the ends of the diameter of a circle to its circumference form a right angle. So in the third diagram, c is a right angle.
A tangent to a circle forms a right angle with the circle's radius, at the point of contact of the tangent (a tangent to a circle is a line that touches the circumference at one point only).
The final diagram shows the alternate segment theorem. In short, the red angles are equal to each other and the green angles are equal to each other.

A cyclic quadrilateral is a four sided figure in a circle, with each vertex (corner) of the quadrilateral touching the circumference of the circle. The opposite angles of a such a quadrilateral add up to 180 degrees.

Area of Sector and Arc Length



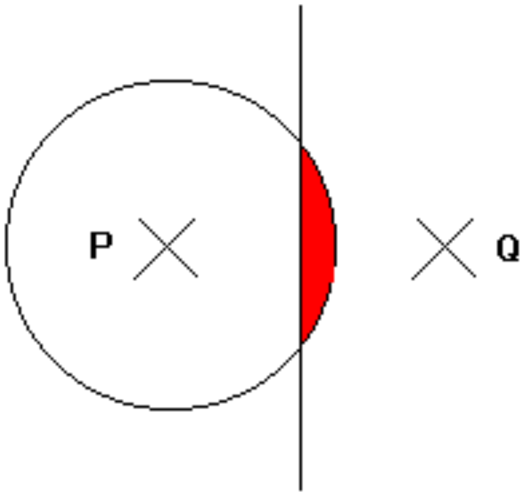
If the radius of the circle is r,
Area of sector = $pr^2 \times A/360$
Arc length = $2pr \times A/360$

In other words, area of sector = area of circle × A/360
arc length = circumference of circle × A/360

Loci

Loci
A locus is a set of points satisfying a certain condition. The term 'locus', however, is rarely used in exams. The question is more likely to be of this format:

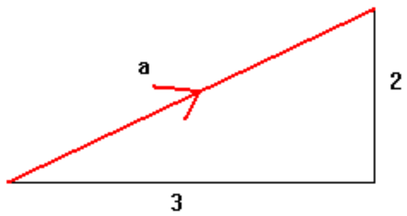
Example:
The diagram shows two points P and Q. On the diagram shade the region which contains all the points which satisfy both the following: the distance from P is less than 3cm, the distance from P is greater than the distance from Q.



All of the points on the circumference of the circle are 3cm from P. Therefore all of the points satisfying the condition that the distance from P is less than 3cm are in the circle.
If we draw a line in the middle of P and Q, all of the points on this line will be the same distance from P as they are from Q. They will be therefore closer to Q, and further away from P, if they are on the right of such a line.
Therefore all of the points satisfying both of these conditions are shaded in red.

Vectors

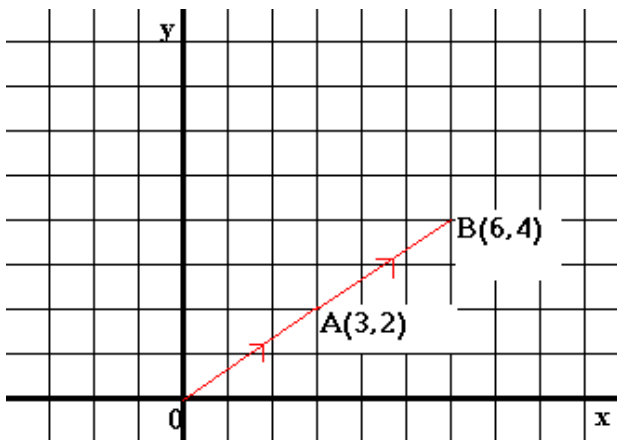
Vectors
A vector quantity has both length (magnitude) and direction. The opposite is a scalar quantity, which only has magnitude. Vectors can be denoted by AB, a, or AB (with an arrow above the letters).
If a = $\begin{pmatrix}3\\2\end{pmatrix}$ then the vector will look as follows:



NB1: When writing vectors as one number above another in brackets, this is known as a column vector.
NB2: in textbooks and here, vectors are indicated by bold type. However, when you write them, you need to put a line underneath the vector to indicate it.

Multiplication by a Scalar
When multiplying a vector by a scalar (i.e. a number), multiply each component of the vector by the scalar.

Example:
If a = $\begin{pmatrix}3\\2\end{pmatrix}$, and b = 2a, sketch a and b.
If a = $\begin{pmatrix}3\\2\end{pmatrix}$, 2a = $\begin{pmatrix}6\\4\end{pmatrix}$



Vector Manipulation

$a = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $b = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$2a = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ $a + b = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$

$-a = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

The length, or modulus, of a vector is denoted by |a| . It can be found by Pythagoras' theorem.

If $a = \begin{pmatrix} x \\ y \end{pmatrix}$ $|a| = \sqrt{(x^2 + y^2)}$

If a and b are parallel vectors, a = kb, where k is scalar

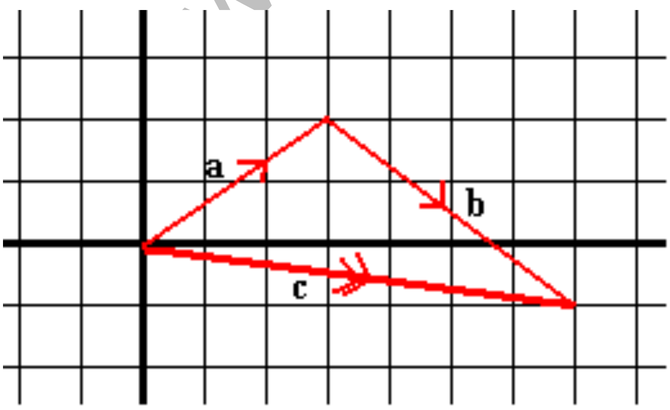
Example:

If a = (-5) and b = (2), find the magnitude of their resultant.
(3) (1)

The resultant of two or more vectors is their sum.
The resultant therefore is (-3).
(4)
The magnitude of this is Ö(-3² + 4²) = Ö(9 + 16) = Ö(25) = 5

The addition and subtraction of vectors can be shown diagrammatically. To find **a + b**, draw **a** and then draw **b** at the end of **a**. The resultant is the line between the start of **a** and the end of **b.**
To find **a - b**, find **-b** (see above) and add this to **a**.

Example:



a = (3) b = (4)
(2) (- 3)

Find a + b

Answer: c = (7)
(-1)

Unit Vectors

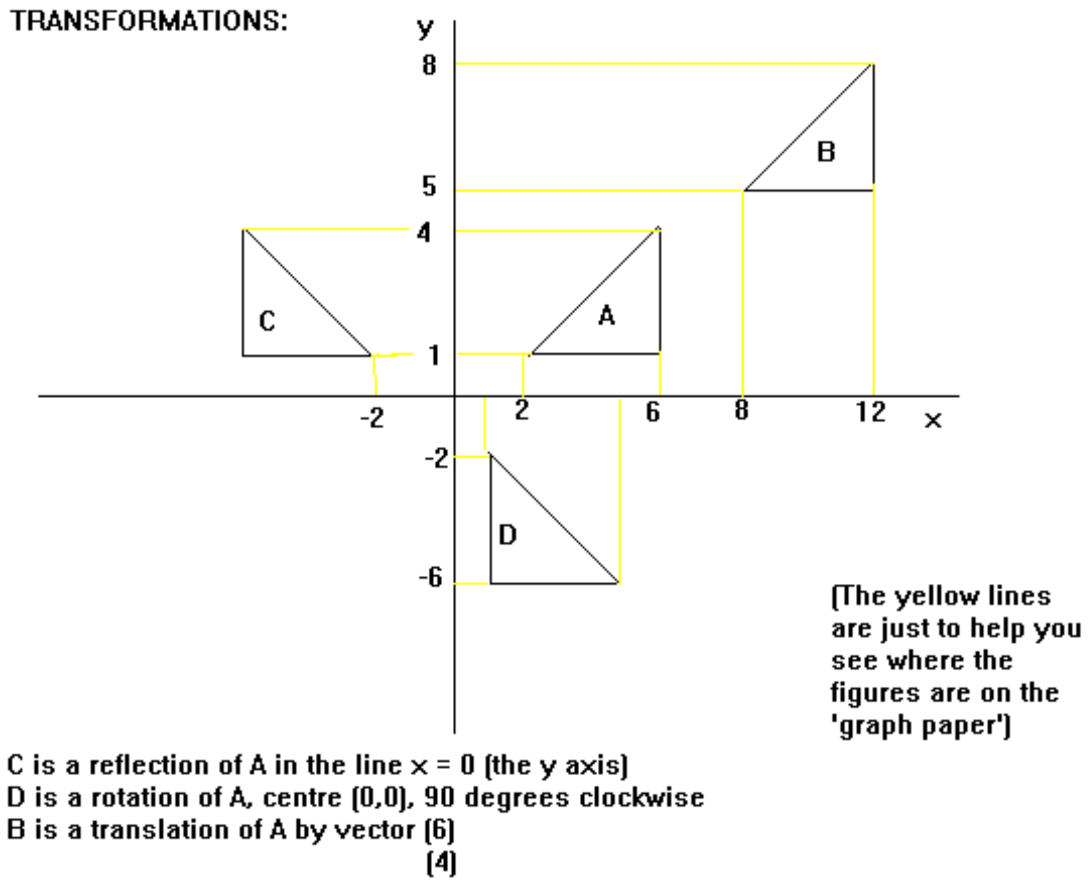
A unit vector has a magnitude of 1. The unit vector in the direction of the x-axis is i and the unit vector in the direction of the y-axis is j. For example on a graph, 3i + 4j would be at (3 , 4). This method is another method of writing down vectors.

Example: 3i + j plus 5i - 4j = 8i - 3j. This is equivalent to:

$$\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} 8 \\ -3 \end{pmatrix}$$

Transformations

Transformations
A translation occurs when a shape has been moved from one place to another. It is equivalent of picking up the shape and putting it down somewhere else. Vectors are used to describe such transformations.
When describing a reflection, you need to state the line which the shape has been reflected in.
When describing a rotation, the centre and angle of rotation are given. If you wish to use tracing paper to help with rotations: draw the shape you wish to rotate onto the tracing paper and put this over shape. Push the end of your pencil down onto the tracing paper, where the centre of rotation is and turn the tracing paper through the appropriate angle. Where the shape is on the tracing paper is where the rotated version goes.



Enlargements

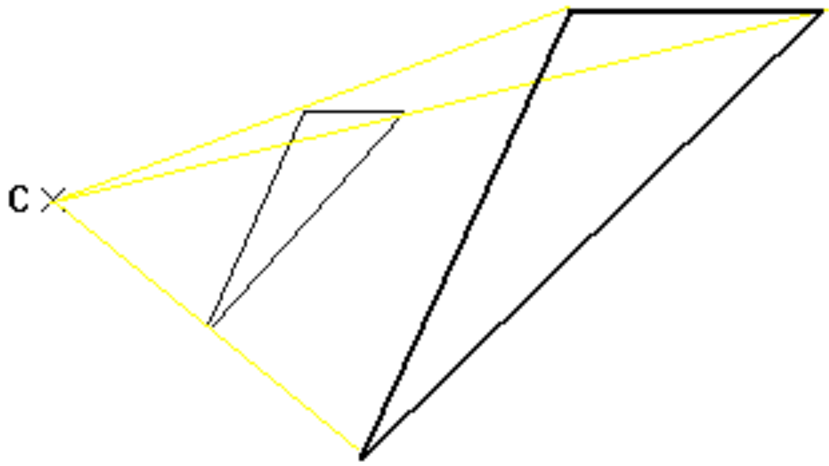
Enlargements have a centre of enlargement and a scale factor.

1) Draw a line from the centre of enlargement to each vertex ('corner') of the shape you wish to enlarge. Measure the lengths of each of these lines.

2) If the scale factor is 2, draw a line from the centre of enlargement, through each vertex, which is twice as long as the length you measured. If the scale factor is 3, draw lines which are three times as long. If the scale factor is 1/2, draw lines which are 1/2 as long.

Example:

The centre of enlargement is marked. Enlarge the triangle by a scale factor of 2.



Good Luck in your Exams!