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**Math(s) = Maths!**

If you are in Singapore you may want to use this link to view your syllabus.

<http://www.seab.gov.sg/oLevel/oLevel.html>

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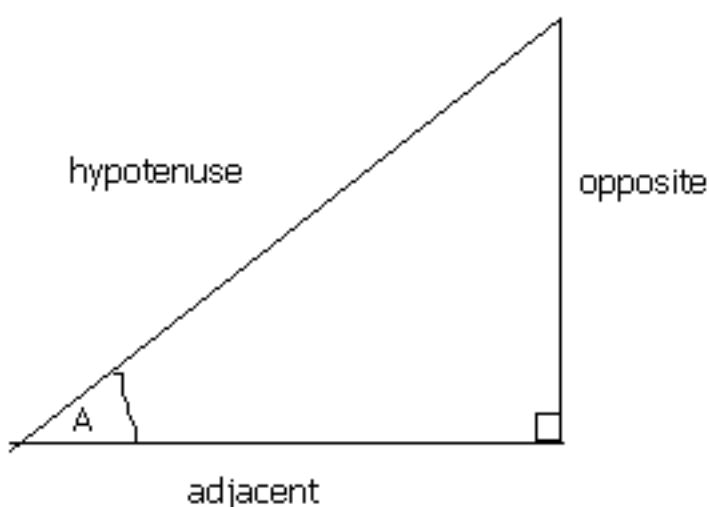
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## Sin, Cos and Tan



The sine of the angle =  $\frac{\text{the length of the opposite side}}{\text{the length of the hypotenuse}}$

The cosine of the angle =  $\frac{\text{the length of the adjacent side}}{\text{the length of the hypotenuse}}$

The tangent of the angle =  $\frac{\text{the length of the opposite side}}{\text{the length of the adjacent side}}$

The hypotenuse of a right angled triangle is the longest side, which is the one opposite the right angle. The adjacent side is the side which is between the angle in question and the right angle. The opposite side is opposite the angle in question.

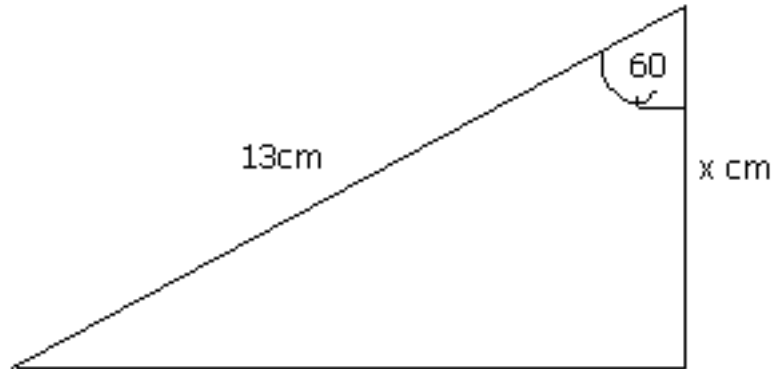
$$\sin = o/h \quad \cos = a/h \quad \tan = o/a$$

Often remembered by: soh cah toa

$$\tan A = \sin A / \cos A$$

*Example:*

Find the length of side  $x$  in the diagram below:



The angle is 60 degrees. We are given the hypotenuse and need to find the adjacent side. This formula which connects these three is:

$\cos(\text{angle}) = \text{adjacent} / \text{hypotenuse}$

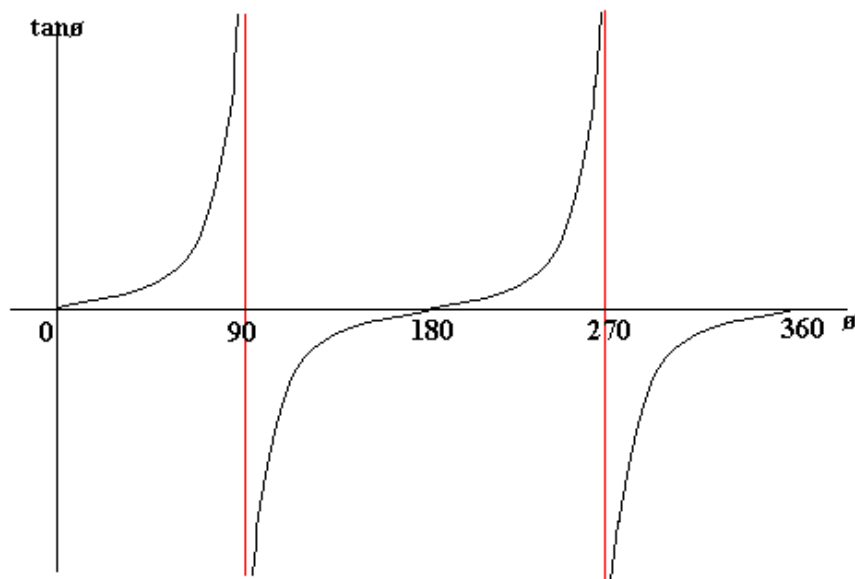
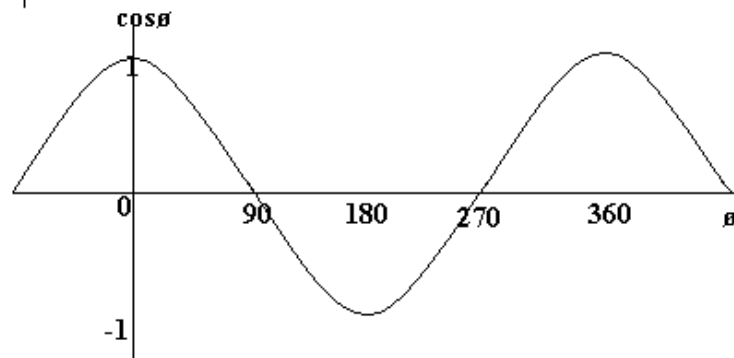
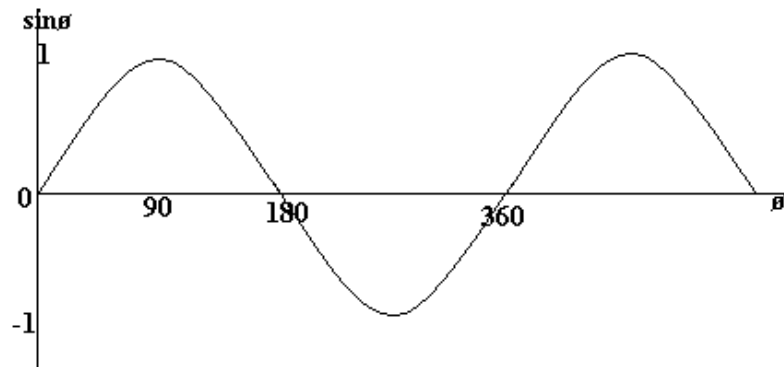
therefore,  $\cos 60 = x / 13$

therefore,  $x = 13 \times \cos 60 = 6.5$

therefore the length of side  $x$  is 6.5cm.

**The graphs of sin, cos and tan:**

The following graphs show the value of  $\sin\theta$ ,  $\cos\theta$  and  $\tan\theta$  against  $\theta$  ( $\theta$  represents an angle). From the sin graph we can see that  $\sin\theta = 0$  when  $\theta = 0$  degrees, 180 degrees and 360 degrees.

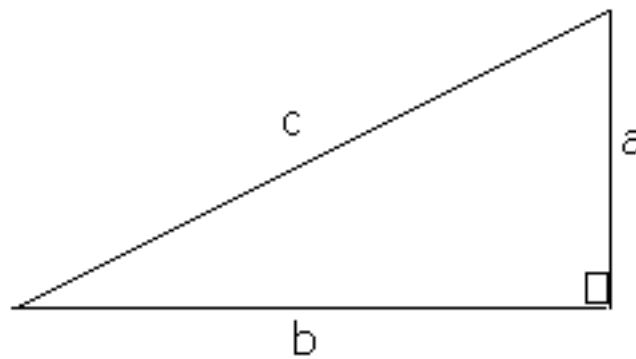


## Pythagoras's Theorem

### Pythagoras's Theorem

In any right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

ie:  $c^2 = a^2 + b^2$



*Example:*

Find AC in the diagram below.

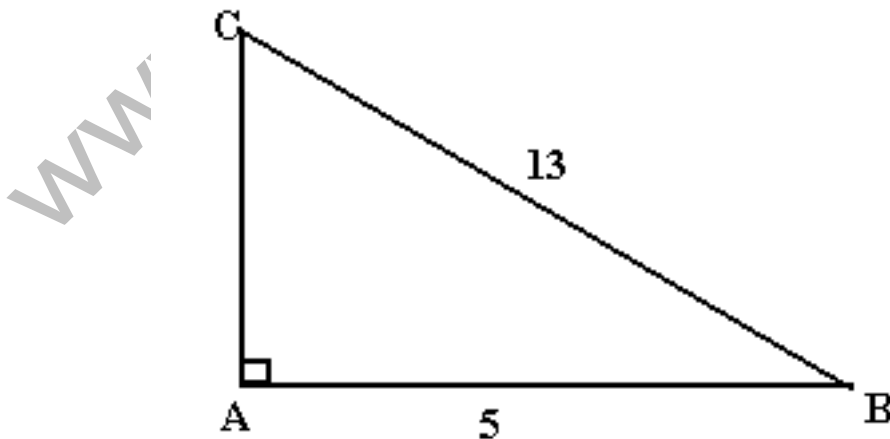
$$AB^2 + AC^2 = BC^2$$

$$AC^2 = BC^2 - AB^2$$

$$= 13^2 - 5^2$$

$$= 169 - 25 = 144$$

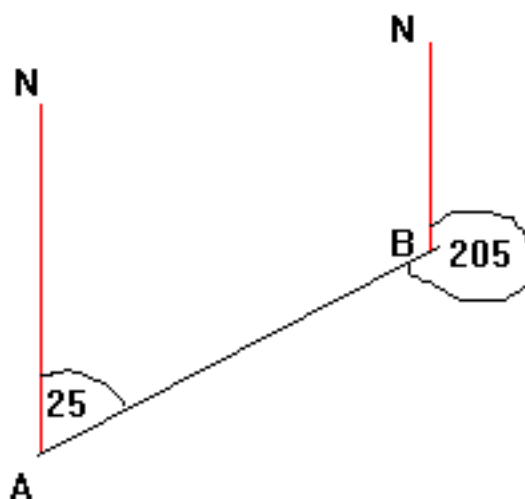
$$AC = \underline{12\text{cm}}$$



## Bearings

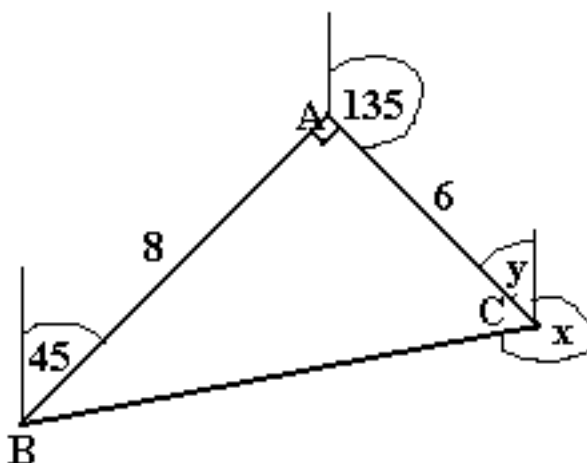
### Bearings

A bearing is the angle, measured clockwise from the north direction. Below, the bearing of B from A is 025 degrees (note 3 figures are always given). The bearing of A from B is 205 degrees.



### Example:

A, B and C are three ships. The bearing of A from B is 045°. The bearing of C from A is 135°. If AB= 8km and AC= 6km, what is the bearing of B from C?



$$\tan C = 8/6, \text{ so } C = 53.13^\circ$$

$$y = 180^\circ - 135^\circ = 45^\circ \text{ (interior angles)}$$

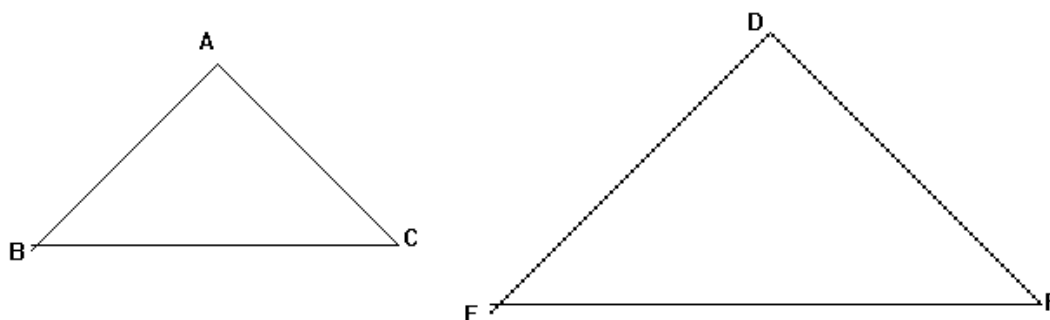
$$x = 360^\circ - 53.13^\circ - 45^\circ \text{ (angles round a point)}$$

$$= \underline{262^\circ} \text{ (to the nearest whole number)}$$

## Similar Triangles

### Similar Triangles

If two shapes are similar, one is an enlargement of the another. This means that the two shapes will have the same angles and their sides will be in the same proportion (e.g. the sides of one triangle will all be 3 times the sides of the other etc.).



angle A = angle D  
angle B = angle E  
angle C = angle F

$AB/DE = BC/EF = AC/DF = \text{perimeter of ABC} / \text{perimeter of DEF}$

Two triangles are similar if:

- 1) 3 angles of 1 triangle are the same as 3 angles of the other
- or 2) 3 pairs of corresponding sides are in the same ratio
- or 3) An angle of 1 triangle is the same as the angle of the other triangle and the sides containing these angles are in the same ratio.

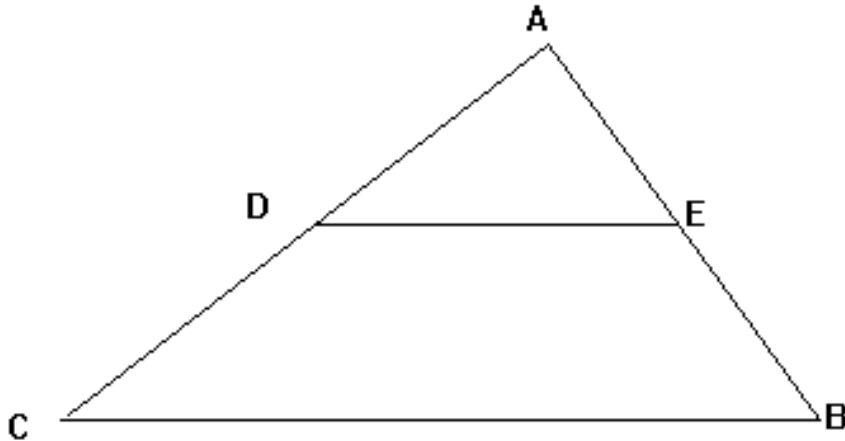
*Example:*

In the above diagram, the triangles are similar.  $EF = 6\text{cm}$  and  $BC = 2\text{cm}$ .

What is the length of  $DE$  if  $AB$  is  $3\text{cm}$ ?

$EF = 3BC$ , so  $DE = 3AB = 9\text{cm}$ .

## Triangle Intercept Theorem



If CB and DE are parallel, the ratio of CD to DA and the ratio of BE to EA are equal. In other words,  $CD/DA = BE/EA$

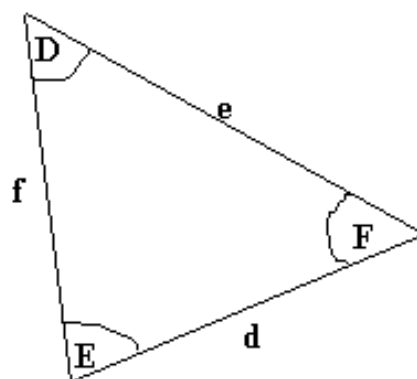
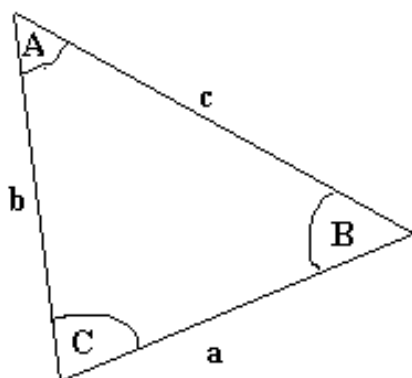


## Congruency

Basically, if two shapes are congruent, they are the same (shape and size). It is often useful to know whether two triangles are congruent.

*Two triangles are congruent if any one of the following is true:*

- All three sides of one triangle are the same length as all three sides of the other triangle (i.e.  $a = d$ ,  $b = f$  and  $c = e$  below).
- Two of the angles and a side of one triangle are equal to the corresponding two angles and side of the other triangle (e.g.  $A = D$ ,  $C = E$  and  $a = d$ ).
- An angle between two sides of a triangle is equal to the corresponding angle in the other triangle and the sides in question are equal (e.g.  $C = E$ ,  $b = f$ ,  $a = d$ ).
- Two right angled triangles have the same hypotenuse and one other equal side.



## Sine and Cosine Formulae

Sine and Cosine Formulae

Sine and Cosine Formulae:

$$\sin x = \sin (180 - x)$$

e.g.  $\sin 130 = \sin (180 - 130) = \sin 50$

$$\cos x = -\cos (180 - x)$$

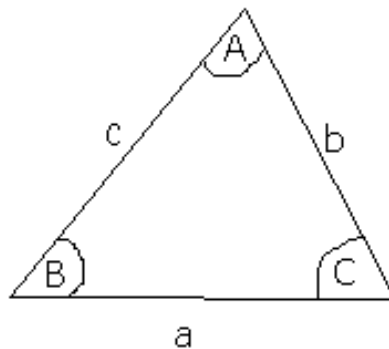
*The Sine Rule:*

This works in any triangle:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

alternatively,  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

NOTE: the triangle is labelled as follows:



*The Cosine Rule:*

$$c^2 = a^2 + b^2 - 2ab\cos C$$

can also be written as:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

This also works in any triangle.

### The area of a triangle

The area of any triangle is  $\frac{1}{2} ab\sin C$  (using the above notation)

This formula is useful if you don't know the height of a triangle (since you need to know the height for  $\frac{1}{2}$  base  $\times$  height).

**Good Luck in your Exams!**